

CHAPTER 5. DISCRETE PROBABILITY AND DISTRIBUTIONS

SAMPLE TEST

1) Determine whether each of the following variables is discrete or continuous

- a) weight of a fish
- b) distance between road signal
- c) number of road signals per mile
- d) area of a parking lot
- e) cars in a parking lot
- f) books in a shelf
- g) pages in a book
- h) temperature in our classroom

2) Determine if the following table represents a probability distribution

X	2	4	5	8	12
P(X)	0.35	0.25	0.30	0.08	0.02

3) Determine if the following table represents a probability distribution

X	1	2	3	4
P(X)	0.48	0.22	0.30	0.12

4) Determine if the following table represents a probability distribution

X	0.1	0.3	0.5	0.7
P(X)	0.28	0.38	- 0.36	0.70

5) A company tested a new flashlight and found that the number of errors had the following probability distribution. Find the mean number of errors.

Number of errors	2	3	4	5	6
P(X)	0.02	0.25	0.40	0.30	0.03

6) Find the variance and the standard deviation of the product tested on question 5.

STATISTICS

7) Find the mean, variance and standard deviation of the following probability distribution.

X	-5	6	9
P(X)	0.5	0.25	0.25

8) Find the mean, variance and standard deviation of the following probability distribution.

Number of errors	0	1	2	3	4
P(X)	0.05	0.20	0.50	0.20	0.05

9) Construct a probability distribution for the number of absences per day in April at PMW office.

Number of absences	0	1	2	3	4	5	6
Frequency	11	6	6	3	3	0	1

BINOMIAL DISTRIBUTION

10) According to his experience Eduard has 80% chance of win playing in a certain cards game What is the mean number of times that he will win when he plays 20 times?

11) Find the variance and standard deviation of wins on problem 10.

12) If the probability of rain on any given day on April in Tacoloya County is 40%, find the mean, variance and the standard deviation of number of days of rain.

13) A basketball player has 80% chance of making a foul shot. If he shoots 8 fouls, what is the probability that he makes exactly 6?

14) A basketball player has 60% chance of making a foul shot. If he shoots 10 fouls, what is the probability that he makes at least 8 shots?

15) TJ is going to guess in a true or false test that has 10 questions. What is the probability that he gets exactly 7 correct answers?

- 16) What is the mean number of correct answers that TJ will get on problem 15.
- 17) The probability to catch a fish early in the morning on any given day is 35%. If you are going to go fishing 7 consecutive days, what is the probability to catch a fish exactly 4 days?
- 18) A multiple-choice test has 10 questions each of which has 5 possible answers, only one which is correct. Juliet, who forgot to study for the test, guesses on all questions, what is the probability that she will answer exactly 3 questions correctly?
- 19) A multiple-choice test has 12 questions each of which has 5 possible answers, only one which is correct. Romeo, who forgot to study for the test, guesses on all questions, what is the probability that he will answer at most 3 questions correctly?
- 20) A company manufactures light bulbs in batches of 200 and there are 5% rate of defects. Find the mean and standard deviation number of light bulbs without defects per batch.
- 21) Find the mean and the standard deviation of a Binomial Distribution with $n = 60$ and $p = 25\%$.
- 22) Find the mean and the standard deviation of a Binomial Distribution with $n = 25$ and $q = 80\%$.
- 23) Six percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 50 parts contains exactly 3 defective ones?
- 24) A fair coin is tossed 100 times. What is the probability to get exactly 50 heads?
- 25) A fair six-sided die is rolled 100 times. What is the probability to get exactly 40 numbers greater than 4?
- 26) A pair of fair six-sided dice is rolled 50 times. What is the probability to roll exactly 5 times a sum of 7?

- 27) A pair of fair six-sided dice is rolled 50 times. What is the probability to roll exactly 5 times a sum of 12?
- 28) By some estimates, twenty-percent (20%) of the residents in ABC county have no health insurance. Randomly sample $n = 15$ residents. What is the probability that exactly 5 of the 15 sampled have no health insurance?
- 29) On problem 28, what is the probability that at most 5 residents have health insurance?
- 30) The passing grade in a **true or false** test is 70%. Tom, who forgot to study for the test will guess all the questions. What is the probability that he will pass the test if the requirement to pass is to answer at least 7 correct answers?

Formulas and the answers for all the exercises in the next pages

FORMULAS

Mean, Variance and Standard Deviation of a Probability Distribution

$\mu = \sum [x.P(x)]$	Mean
$\sigma^2 = \sum [(x - \mu)^2 . P(x)] = \sum [x^2 . P(x)] - \mu^2$	Variance
$\sigma = \sqrt{\sum [(x - \mu)^2 . P(x)]} = \sqrt{\sum [x^2 . P(x)] - \mu^2}$	Standard Deviation

Mean, Variance and Standard Deviation of a Binomial Distribution

$\mu = np$	Mean
$\sigma^2 = npq$	Variance
$\sigma = \sqrt{npq}$	Standard Deviation
Where:	
n : number of fixed trials	
p : probability of success in one of the n trials	
q : probability of failure in one of the n trials	

Binomial Distribution Formula

$P(x) = \frac{n!}{(n-x)!x!} p^x q^{n-x}$
Where:
n : number of fixed trials
X : total number of successes
p : probability of success in one of the n trials
q : probability of failure in one of the n trials

ANSWERS

1) Continuous: a, b, d, and h

Discrete: c, e, f, and g

2) Yes, it is a probability distribution, since all the probabilities are between 0 and 1, and they add to 1

3) This is not a probability distribution, since all the probabilities add to 1.12 not 1

4) This is not a probability distribution, since $P(0.5) = -0.36$, and probabilities cannot be negative

5) Mean = $2(0.02) + 3(0.25) + 4(0.40) + 5(0.30) + 6(0.03) = 4.07$

6) Variance = $[(2)^2(0.02) + (3)^2(0.25) + (4)^2(0.40) + (5)^2(0.30) + (6)^2(0.03)] - (4.07)^2 = 0.74$

Standard deviation = $\sqrt{0.74} = 0.86$

7) Mean = $-5(0.5) + 6(0.25) + 9(0.25) = 1.25$

Variance = $[(-5)^2(0.5) + (6)^2(0.25) + (9)^2(0.25)] - (1.25)^2 = 40.19$

Standard Deviation = $\sqrt{40.19} = 6.34$

8) Mean = $0(0.05) + 1(0.20) + 2(0.50) + 3(0.20) + 4(0.05) = 2.00$

Variance = $(0)^2(0.05) + (1)^2(0.20) + (2)^2(0.50) + (3)^2(0.20) + (4)^2(0.05) = 0.80$

Standard Deviation = $\sqrt{0.80} = 0.89$

9) $P(x) = \text{Frequency of } X / \text{Sum of the frequencies}$

Number of absences (X)	0	1	2	3	4	5	6
P (X)	0.37	0.20	0.20	0.10	0.10	0.00	0.03

10) Mean $\mu = np = 20(0.80) = 16$

11) Variance $\sigma^2 = npq = 20(0.80)(0.20) = 3.2$

Standard deviation $\sigma = \sqrt{npq} = \sqrt{3.2} = 1.8$

12) Mean $\mu = np = 30(0.40) = 12$ days of rain ($n = 30$ because April has 30 days)

Variance $\sigma^2 = npq = 30(0.40)(0.60) = 7.2$

Standard deviation $\sigma = \sqrt{npq} = \sqrt{7.2} = 2.7$

13) $n = 8$, $p = 80\% = 0.80$, $q = (1 - p) = 0.20$

$$P(6) = \frac{8!}{(8-6)! \cdot 6!} (0.80)^6 (0.20)^{8-6} = 0.2936 = 29.36\%$$

14) $n = 10$, $p = 60\% = 0.60$, $q = (1 - p) = 0.40$

$P(\text{at least } 8) = P(X = 8) + P(X = 9) + P(X = 10)$

$P(\text{at least } 8) = 0.1209 + 0.0403 + 0.0060 = 0.1672 = 16.72\%$

<i>n</i>	<i>x</i>	<i>p</i>										
		0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
10	0	0.599	0.349	0.107	0.028	0.006	0.001					
	1	0.315	0.387	0.268	0.121	0.040	0.010	0.002				
	2	0.075	0.194	0.302	0.233	0.121	0.044	0.011	0.001			
	3	0.010	0.057	0.201	0.267	0.215	0.117	0.042	0.009	0.001		
	4	0.001	0.011	0.088	0.200	0.251	0.205	0.111	0.037	0.006		
	5		0.001	0.026	0.103	0.201	0.246	0.201	0.103	0.026	0.001	
	6			0.006	0.037	0.111	0.205	0.251	0.200	0.088	0.011	0.001
	7			0.001	0.009	0.042	0.117	0.215	0.267	0.201	0.057	0.010
	8				0.001	0.011	0.044	0.121	0.233	0.302	0.194	0.075
	9					0.002	0.010	0.040	0.121	0.268	0.387	0.315
	10						0.001	0.006	0.028	0.107	0.349	0.599

15) $n = 10$,

$X = 7$

$p = 50\% = 0.50$, (in a true/false questions the probability to success is 50%)

$q = (1 - p) = 0.50$

$$P(7) = \frac{10!}{(10-7)! \cdot 7!} (0.50)^7 (0.50)^{10-7} = 0.1172$$

16) Mean number of correct answers: $\mu = np = 10(0.5) = 5$

17) $n = 7$

$X = 4$

$p = 35\% = 0.35, \quad q = 1 - p = 0.65$

$$P(x = 4) = \frac{7!}{(7-4)!4!} (0.35)^4 (0.65)^{7-4} = 0.1442 = 14.42\%$$

18) $n = 10, X = 3$

$p = 1/5 = 0.20$

(only 1 out of the 5 choices is correct)

$q = 1 - p = 0.80$

From the table $P(3) = 0.2013 = 20.13\%$

<i>n</i>	<i>x</i>	<i>p</i>										
		0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
10	0	0.599	0.349	0.107	0.028	0.006	0.001					
	1	0.315	0.387	0.268	0.121	0.040	0.010	0.002				
	2	0.075	0.194	0.302	0.233	0.121	0.044	0.011	0.001			
	3	0.010	0.057	0.201	0.267	0.215	0.117	0.042	0.009	0.001		
	4	0.001	0.011	0.088	0.200	0.251	0.205	0.111	0.037	0.006		
	5		0.001	0.026	0.103	0.201	0.246	0.201	0.103	0.026	0.001	
	6			0.006	0.037	0.111	0.205	0.251	0.200	0.088	0.011	0.001
	7			0.001	0.009	0.042	0.117	0.215	0.267	0.201	0.057	0.010
	8				0.001	0.011	0.044	0.121	0.233	0.302	0.194	0.075
	9					0.002	0.010	0.040	0.121	0.268	0.387	0.315
	10						0.001	0.006	0.028	0.107	0.349	0.599

19) $n = 12,$

$X = 0, 1, 2,$ and 3

$p = 1/5 = 0.20, \quad q = 1 - p = 0.80$

(only 1 out of the 5 choices is correct)

$P(\text{at most 3 correct answers}) = P(0) + P(1) + P(2) + P(3)$

$P(\text{at most 3 correct answers}) = 0.069 + 0.206 + 0.283 + 0.236 = 0.7945 = 79.45\%$

<i>n</i>	<i>x</i>	<i>p</i>										
		0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
12	0	0.540	0.282	0.069	0.014	0.002						
	1	0.341	0.377	0.206	0.071	0.017	0.003					
	2	0.099	0.230	0.283	0.168	0.064	0.016	0.002				
	3	0.017	0.085	0.236	0.240	0.142	0.054	0.012	0.001			
	4	0.002	0.021	0.133	0.231	0.213	0.121	0.042	0.008	0.001		
	5		0.004	0.053	0.158	0.227	0.193	0.101	0.029	0.003		
	6			0.016	0.079	0.177	0.226	0.177	0.079	0.016		
	7			0.003	0.029	0.101	0.193	0.227	0.158	0.053	0.004	
	8			0.001	0.008	0.042	0.121	0.213	0.231	0.133	0.021	0.002
	9				0.001	0.012	0.054	0.142	0.240	0.236	0.085	0.017
	10					0.002	0.016	0.064	0.168	0.283	0.230	0.099
	11						0.003	0.017	0.071	0.206	0.377	0.341
	12							0.002	0.014	0.069	0.282	0.540

20) $n = 200$

$p = 95\% = 0.95$ (probability of light bulbs without defects)

$q = 1 - p = 0.05$

Mean: $\mu = np = 200(0.95) = 190$

Variance: $\sigma^2 = npq = 200(0.95)(0.05) = 9.5$

Standard deviation $\sigma = \sqrt{npq} = \sqrt{9.5} = 3.1$

21) $n = 60$

$p = 25\% = 0.25$

$q = 1 - p = 0.75$

Mean: $\mu = np = 60(0.25) = 15$

Variance: $\sigma^2 = npq = 60(0.25)(0.75) = 11.25$

Standard deviation $\sigma = \sqrt{npq} = \sqrt{11.25} = 3.35$

22) $n = 25$

$q = 80\% = 0.80$

$p = 1 - q = 0.20$

Mean: $\mu = np = 25(0.20) = 5$

Variance: $\sigma^2 = npq = 25(0.20)(0.80) = 4.0$

Standard deviation $\sigma = \sqrt{npq} = \sqrt{4} = 2.0$

23) $n = 50$

$X = 3$

$p = 6\% = 0.06$

$q = 1 - p = 0.94$

$$P(\text{exactly } 3) = \frac{50!}{(50-3)!3!} (0.06)^3 (0.94)^{50-3} = 0.2311$$

24) $n = 100$

$X = 50$ $p = 50\% = 0.50$, $q = 1 - p = 0.50$

$$P(\text{exactly } 50) = \frac{100!}{(100-50)!50!} (0.50)^{50} (0.50)^{100-50} = 0.0796 = 7.96\%$$

25) $n = 100$

$X = 40$

$p = 2/6 = 1/3$

(there are 2 numbers greater than 4 in a die)

$q = 1 - p = 2/3$

$$P(\text{exactly 40 numbers} > 4) = \frac{100!}{(100-40)!40!} \left(\frac{1}{3}\right)^{40} \left(\frac{2}{3}\right)^{100-40} = 0.0308 = 3.08\%$$

26) $n = 50$

$X = 5$

$p = 6/36 = 1/6$

(there are 6 combinations of 2 dice to get a sum of 7)

$q = 1 - p = 5/6$

$$P(\text{exactly 5 times a sum of 7}) = \frac{50!}{(50-5)!5!} \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^{50-5} = 0.0745 = 7.45\%$$

27) $n = 50$

$X = 5$

$p = 1/36$

(there is only 1 combinations of 2 dice to get a sum of 12)

$q = 1 - p = 35/36$

$$P(\text{exactly 5 times a sum of 12}) = \frac{50!}{(50-5)!5!} \left(\frac{1}{36}\right)^5 \left(\frac{35}{36}\right)^{50-5} = 0.0099 = 0.99\%$$

28) $n = 15$

$X = 5$

$p = 20\% = 0.20$

$q = 1 - p = 0.80$

$$P(\text{exactly 5 have no health insurance}) = \frac{15!}{(15-5)!5!} (0.20)^5 (0.80)^{15-5} = 0.1032 = 10.32\%$$

29) $n = 15$

$X = 0, 1, 2, 3, 4, 5$

$p = 20\% = 0.20$, $q = 1 - p = 0.80$

$P(\text{at most } 5) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

From the Binomial Distribution Table:

$P(\text{at most } 5) = 0.0352 + 0.1319 + 0.2309 + 0.2501 + 0.1876 + 0.1032 = 0.9389 = 93.89\%$

n	x	p										
		0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
15	0	0.463	0.206	0.035	0.005							
	1	0.366	0.343	0.132	0.031	0.005						
	2	0.135	0.267	0.231	0.092	0.022	0.003					
	3	0.031	0.129	0.250	0.170	0.063	0.014	0.002				
	4	0.005	0.043	0.188	0.219	0.127	0.042	0.007	0.001			
	5	0.001	0.010	0.103	0.206	0.186	0.092	0.024	0.003			
	6		0.002	0.043	0.147	0.207	0.153	0.061	0.012	0.001		
	7			0.014	0.081	0.177	0.196	0.118	0.035	0.003		
	8			0.003	0.035	0.118	0.196	0.177	0.081	0.014		
	9			0.001	0.012	0.061	0.153	0.207	0.147	0.043	0.002	
	10				0.003	0.024	0.092	0.186	0.206	0.103	0.010	0.001
	11				0.001	0.007	0.042	0.127	0.219	0.188	0.043	0.005
	12					0.002	0.014	0.063	0.170	0.250	0.129	0.031
	13						0.003	0.022	0.092	0.231	0.267	0.135
	14							0.005	0.031	0.132	0.343	0.366
	15								0.005	0.035	0.206	0.463

30) $n = 10$

$X = 7, 8, 9, 10$

(Tom needs 7 or more correct answers to pass the test)

$p = 50\% = 0.50$

$q = 1 - p = 0.50$

$P(\text{at least } 7 \text{ correct answers}) = P(7) + P(8) + P(9) + P(10)$

From the Binomial Distribution Table:

$P(\text{at least } 7 \text{ correct answers}) = 0.1172 + 0.0439 + 0.00098 + 0.0001 = 0.1719 = 17.19\%$

n	x	p										
		0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
10	0	0.599	0.349	0.107	0.028	0.006	0.001					
	1	0.315	0.387	0.268	0.121	0.040	0.010	0.002				
	2	0.075	0.194	0.302	0.233	0.121	0.044	0.011	0.001			
	3	0.010	0.057	0.201	0.267	0.215	0.117	0.042	0.009	0.001		
	4	0.001	0.011	0.088	0.200	0.251	0.205	0.111	0.037	0.006		
	5		0.001	0.026	0.103	0.201	0.246	0.201	0.103	0.026	0.001	
	6			0.006	0.037	0.111	0.205	0.251	0.200	0.088	0.011	0.001
	7			0.001	0.009	0.042	0.117	0.215	0.267	0.201	0.057	0.010
	8				0.001	0.011	0.044	0.121	0.233	0.302	0.194	0.075
	9					0.002	0.010	0.040	0.121	0.268	0.387	0.315
	10						0.001	0.006	0.028	0.107	0.349	0.599